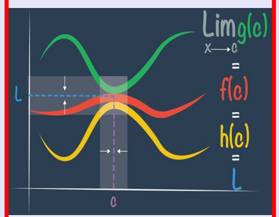
Calculus I

Lecture 37



Feb 19-8:47 AM

$$f(x) = \frac{x^3}{x+8}$$

Domain $x \neq -8 \rightarrow$ V.A. $x = -8$

Y-Int $(0,0)$ x-Int $f(x)=0$ $\frac{x^3}{x+8} = 0$ $x^3 = 0$ $\boxed{x=0}$ odd # of times

If $\frac{x}{x+8} \rightarrow$ H.A. $y = 1$

If $\frac{x^2}{x+8} \rightarrow$ slant Asymptote

$f(x) = \frac{x^3}{x+8} \rightarrow$ Quadratic

Long Division

$$\begin{array}{r} x^2 \quad -8x \quad +64 \\ x+8 \,\big)\, \overline{x^3 \quad +0x^2 \quad +0x \quad +0} \\ -(x^3 \quad +8x^2 \qquad\qquad) \\ \hline -8x^2 \quad +0x \quad +0 \\ -(-8x^2 \quad -64x \quad) \\ \hline 64x \quad +0 \\ -(64x \quad +512) \\ \hline -512 \end{array}$$

$$f(x) = x^2 - 8x + 64 + \frac{-512}{x+8}$$

$$f'(x) = 2x - 8 + \frac{512}{(x+8)^2} = \boxed{\frac{2x^2(x+12)}{(x+8)^2}}$$

$$f''(x) = 2 - \frac{1024}{(x+8)^3} = \boxed{\frac{2(x+8)^3 - 1024}{(x+8)^3}} = \frac{2x(x^2+24x+192)}{(x+8)^3}$$

C.P. $f'(x) = 0$ $2x^2(x+12) = 0 \rightarrow x = 0$ $x: -12$

$f'(x)$ und. $(x+8)^2 = 0 \rightarrow x = -8$

P.I.P. $f''(x) = 0 \rightarrow 2(x+8)^3 - 1024 = 0$

$f''(x)$ und. $\rightarrow (x+8)^3 = 0 \rightarrow x = -8$

$(x+8)^3 = 512$ $x+8 = \sqrt[3]{512}$

$x+8 = 8$

$x = 0$

Oct 31-7:49 AM

$$f'(x) = \frac{2x^2(x+12)}{(x+8)^2} \qquad f''(x) = \frac{2x(x^2+24x+192)}{(x+8)^3}$$

$$f''(x) = \frac{2x(x^2+24x+144-144+192)}{(x+8)^3}$$

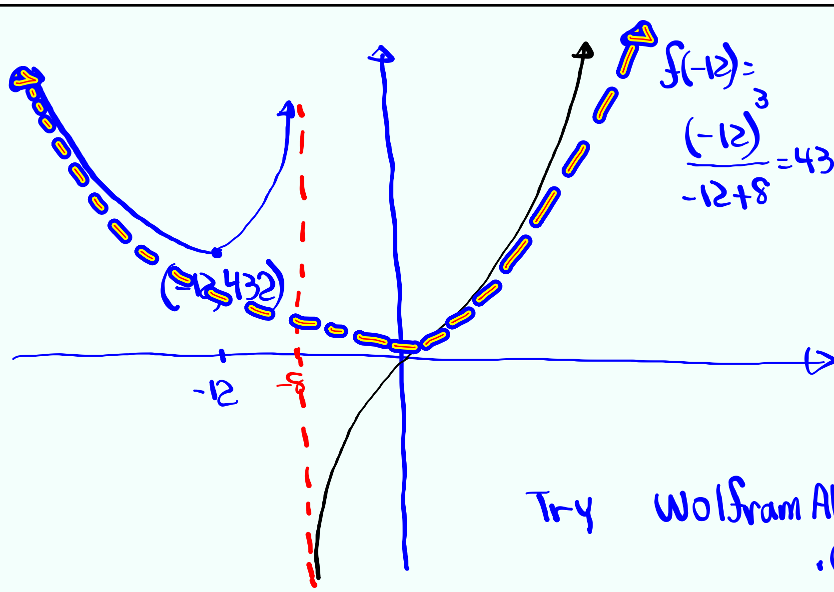
$$f''(x) = \frac{2x((x+12)^2+48)}{(x+8)^3}$$

x	$-\infty$		-12		-8		0		∞
$f'(x)$		$-$	$\bullet$	$+$	$\circ$	$+$	$\bullet$	$+$	
$f''(x)$		$+$		$+$	$\circ$	$-$	$\bullet$	$+$	
$f(x)$		$\searrow$		$\nearrow$		$\nearrow$		$\nearrow$	

Min at $x=-12$

Inflection Point at $x=0$

Nov 5-7:33 AM



$f(-12) = \dfrac{(-12)^3}{-12+8} = 432$

Try WolframAlpha.com

Nov 5-7:45 AM

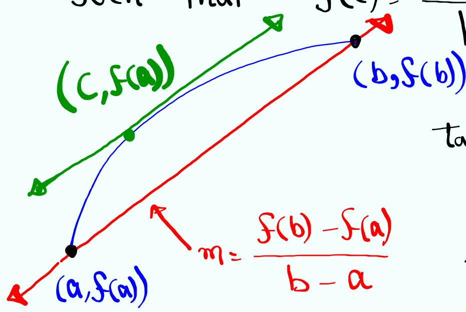
Mean - Value Thrm

$f(x)$ is cont. on $[a,b]$

$f(x)$ is diff. on (a,b)

then there is at least a number c in (a,b)

Such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.



tan. line || Secant line

Same slope

$m_{\text{tan. line}} = m_{\text{sec. line}}$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Nov 4-8:12 AM

$f(x) = \dfrac{1}{x}$, $[1,3]$

Cont. $(-\infty,0)\cup(0,\infty)$

$f'(x) = \dfrac{-1}{x^2}$

undefined at 0.

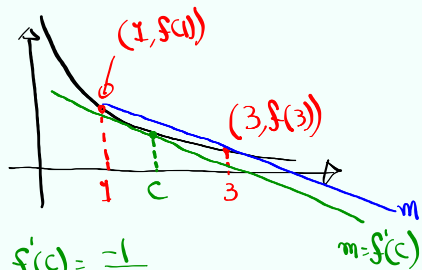
$f(x)$ is Cont. on $[1,3]$

$f(x)$ is diff. on $(1,3)$

there is at least a number c in $(1,3)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$m_{\text{tan. line}} = m_{\text{Secant line}}$



$m = \dfrac{f(3)-f(1)}{3-1}$

$f'(c) = \dfrac{-1}{c^2}$

$$\frac{-1}{c^2} = \frac{\frac{1}{3}-1}{3-1}$$

$$\frac{-1}{c^2} = \frac{-\frac{2}{3}}{2} \qquad \frac{-1}{c^2} = \frac{-1}{3}$$

$c^2 = 3 \qquad c = \sqrt{3}$

$\sqrt{3}$ is in $(1,3)$

Nov 5-7:50 AM

$f(x) = 2x^2 - 3x + 1 \quad , \quad [0,2]$

$f(x)$ is a polynomial $\rightarrow$ Polynomials are cont. & diff. $(-\infty, \infty)$

$f'(x) = 4x - 3$

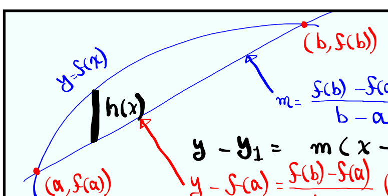
There is at least a number c in $(0,2)$

Such that $\quad f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$$4c - 3 = \frac{f(2) - f(0)}{2 - 0} \qquad 4c - 3 = \frac{3 - 1}{2}$$

$$4c - 3 = 1 \qquad \boxed{c = 1}$$

Nov 5-7:57 AM



$y - y_1 = m(x - x_1)$

$y - f(a) = \dfrac{f(b) - f(a)}{b - a}(x - a)$

$y = \dfrac{f(b) - f(a)}{b - a}(x - a) + f(a)$

$h(x)$ is a function representing vertical distance between Curve & Secant line.

$h(x) = f(x) - \left[\dfrac{f(b) - f(a)}{b - a}(x - a) + f(a)\right]$

$h(a) = f(a) - \left[\dfrac{f(b) - f(a)}{b - a}\underbrace{(a - a)}_{0} + f(a)\right] = f(a) - f(a) = 0$

$h(b) = f(b) - \left[\dfrac{f(b) - \cancel{f(a)}}{\cancel{b - a}}\cancel{(b - a)} + \cancel{f(a)}\right] = f(b) - f(b) = 0$

$h(x) =$ Curve $-$ line

Cont. & diff.

$f(x)$ Conditions of MVT is $f(x)$ is diff. and cont.

$h(x)$ is cont. on $[a,b]$, $h(x)$ is diff on (a,b)

$h(a) = h(b)$

Rolle's Thrm

$h'(z) = 0$ on (a,b)

Nov 5-8:01 AM

$$h(x) = f(x) - \left[\frac{f(b) - f(a)}{b-a}(x-a) + f(a)\right]$$

$$h(x) = f(x) - \frac{f(b) - f(a)}{b-a}(x-a) - f(a)$$

By Rolle's Thrm, $h'(x) = 0$

$$h'(x) = f'(x) - \frac{f(b)-f(a)}{b-a} \cdot 1 - 0$$

$$h'(x) = 0 \rightarrow f'(x) - \frac{f(b)-f(a)}{b-a} = 0$$

$$\boxed{f'(x) = \frac{f(b) - f(a)}{b-a}}$$

Review it, we will do it again tomorrow.

Conclusion of M.V.T.

Nov 5-8:13 AM

If $f(x)$ is diff. at $x=a$, then $f(x)$ is Cont. at $x=a$.

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

we need to show $\lim_{x \to a} f(x) = f(a)$

$$f(x) = f(x) - f(a) + f(a)$$

$$= \frac{f(x) - f(a)}{x-a} \cdot (x-a) + f(a)$$

$$\boxed{\lim_{x \to a} f(x)} = \lim_{x \to a}\left[\frac{f(x)-f(a)}{x-a} \cdot (x-a) + f(a)\right]$$

$$= \lim_{x \to a} \frac{f(x)-f(a)}{x-a}(x-a) + \lim_{x \to a} f(a)$$

$$= \boxed{\lim_{x \to a} \frac{f(x)-f(a)}{x-a}} \cdot \lim_{x \to a}(x-a) + \lim_{x \to a} f(a)$$

$$= f'(a) \cdot \underbrace{(a-a)}_{0} + f(a)$$

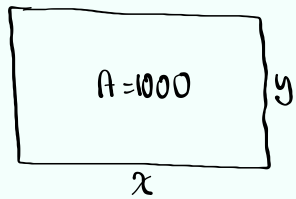
$$= 0 + f(a)$$

$\lim_{x \to a} f(x) = f(a)$ then $f(x)$ is Cont. at $x=a$.

Review it, we do it again.

Nov 5-8:17 AM

Find dimensions of a rectangular garden with minimum perimeter and $1000\ m^2$ area.



$xy = 1000$

$P = 2x + 2y$
Minimum

Let $f(x) = 2x + 2\left(\frac{1000}{x}\right)$

$f(x) = 2x + \frac{2000}{x}$

this has to be minimum

$f'(x)$, $f'(x) = 0$

CU
Min.

$f''(x)$ show concavity at $f'(x) = 0$

Max
CD

Nov 5-8:27 AM

$f'(x) = \boxed{5x^4} - \boxed{3x^2} + \boxed{4}$, $f(-1) = 2$

Find $f(x)$.

$f(x) = x^5 - x^3 + 4x + C$

$f(-1) = (-1)^5 - (-1)^3 + 4(-1) + C = 2$

$-1 + 1 - 4 + C = 2$

$\boxed{C = 6}$

$$\boxed{f(x) = x^5 - x^3 + 4x + 6}$$

Nov 5-8:32 AM